

Towards a constructivist pedagogy for Year 12 mathematics

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Introduction

Constructivist pedagogies are generally not considered to support the teaching of mathematics for externally assessed examination-based courses. In large part, we (teachers) have believed that such approaches are inefficient in covering a set syllabus. This paper summarises my learning journey in Year 12 mathematics in 2004 where attempts were made to use constructivist and inquiry based approaches.

After 30 years as a teacher of mathematics, I had the opportunity to focus on my mathematics teaching in a new school committed to innovative approaches to teaching and learning. This contrasts to my previous position (in a small independent school) where I taught across a wide range of subjects as well as having many other responsibilities. Despite having several students achieve “perfect scores” in their South Australian Certificate of Education (SACE) and others working at world class levels in their chosen field, I felt there was some scope for a real change in what I did and how students learned in my mathematics classes and ultimately throughout their lives.

I feel very fortunate to have had the opportunity to take this learning journey. In particular, I have been able to focus on the Year 12 mathematics courses as my primary responsibility and enjoy the support and encouragement of the mathematics staff. Data has been collected in a reflective journal accompanied by an ongoing discussion with a critical friend, a survey of student attitudes to studying mathematics and a videotape of a session. Furthermore the Australian Science and Mathematics School (ASMS) provides direction for curriculum development in the form of teaching and learning principles. Constructivist and inquiry based approaches are part of these principles.

ASMS

The ASMS is a South Australian Education Department school that caters for the final three years of schooling before entry into higher education. The purpose built school is designed for highly collaborative, interactive and student-directed teaching and learning. The first student cohort in 2003 comprised Year 10 and 11 students with the first group of Year 12s completing their SACE pattern in 2004.

It is not a selective school with the majority of students coming from the local area. Students have a wide variety of abilities and aspirations. The school was founded to provide a strong emphasis on the disciplines of science and mathematics through innovative teaching and learning methodologies. It is located on the campus of Flinders University in Adelaide, South Australia.

The design of the school's learning and physical environments is based on pivotal beliefs about student centred teaching and learning, lifelong learning, the relevance of science and mathematics to the world's future, the interconnectedness of knowledge and the importance of human communication in all its forms. As stated in the ASMS Teaching and Learning Handbook, what we do is characterised by:

- students increasingly directing their own learning and learning independently;
- learning which occurs as part of a process of constructing knowledge;
- learning which involves learners communicating their questions, intuitions, conjectures, reasons, explanations and ideas;
- learning which involves developing knowledge, skills and dispositions to think and act in ways which determine individual effort, the setting of personal goals and self awareness.

It is my contention that such pedagogies can be effective as measured on traditional performance (e.g., Tertiary Education Rank) grounds while allowing greater development of positive attitudes to learning. In this way we can contribute to the development of lifelong learners as is an aim of the school.

Mathematics and Abstract Thinking

In 2003 the Mathematics and Abstract Thinking (MAT) program was implemented under the guidance of Vern Treilibs using constructivist and inquiry-based approaches to mathematics learning. A core and extension set of investigations provide the basis for students to construct their own mathematical understandings. All students in both Years 10 and 11 had access to the same materials which were written around the topics and sub-topics of the SACE stage 1 curriculum statements. By engaging with these investigations students were expected to reflect upon what they had learned and to record their understandings in notebooks, in effect their personal reminders, notes or textbook. The most common working mode was small, informal groups.

The MAT program was organised into units of approximately four weeks duration. Apart from the Introductory unit at the beginning of each semes-

ter, each unit was completed with a Public Presentation Piece. This was an individual student response to one of the extension activities in the unit. Students were expected to write this up so that it could be read by a peer from outside the school. It is word-processed with appropriate typesetting of the mathematics and is submitted electronically.

Why change?

The cohort for the 2004 Year 12 mathematics courses was primarily students from the MAT course, i.e., students who had been learning mathematics with little chalk and talk. Factors which provided the impetus for change included previous observations of substantial student engagement and learning using a constructivist approach in MAT, the teaching and learning principles espoused by the ASMS and my dissatisfaction with a more traditional examination-driven didactic approach.

I began working at the ASMS in August 2003 and I liked the level of student engagement I saw demonstrated by many students in the MAT program. It provided a relaxed informal atmosphere and the students' attitude to doing mathematics was positive. The "Oh, not maths again," comment just did not seem to exist. I found the quality of mathematics evident in student solutions to the investigations exciting and impressive. Consequently, I wanted to undertake the challenge of carrying these attitudes, levels of engagement and quality of mathematical thinking into the exam-based Year 12 mathematics courses.

Not only were the ASMS teaching and learning principles supposed to direct pedagogical practices in my teaching but they are also closely aligned with personal views that I had developed over my career. In particular, I have recognised that students are individuals who learn in different ways and at their own rates. The corollary of this is that the learner is essentially responsible for their own learning. Another theme that has developed has been the desire to teach for understanding, rather than for algorithmic proficiency.

I was also dissatisfied with the textbook approach to teaching mathematics. Previously I had used a textbook as the basis of my course, the source of problems and support material for students. I felt that students were able to develop competence and the ability to solve certain problems in an examination but for too many it was little more than that. I wanted more students to enjoy what they were doing, not just gain a sense of achievement in being able to do Year 12 mathematics. In South Australia, I would be surprised if there is a class that does not use the Haese (Year 12) texts. The approach is traditional: present some theory (theorem or result is shown) then worked example(s) followed by a set of problems. This approach does not reflect a constructivist view of learning (SACSA, 2004; Slee & Shute, 2002). While using such a text does not determine that a traditional didactic approach must be followed, it seems that one is continually drawn back to it.

Making connections: Constructing understandings in Year 12 mathematics

The course was centred on *core investigations*. These written worksheets presented problems which were often embedded in a conversation or story line. They covered the key ideas outlined in the curriculum statement. Extensions offered further investigations for interested students, and it was suggested that students aspiring to an “A” would need to do these. The materials were written with the intention that students would be able to work through these activities by themselves, in line with the idea that students would be able to learn at any time and anywhere. In scheduled classes, students usually worked in small self selected groups and teacher support was available. Such an approach supported students to engage with the material, reflect and seek assistance where required. In this way they were able to construct their own understandings of the mathematics.

After the completion of a core investigation, students were expected to make an entry in their notebooks. These entries were a summary of the key ideas, in effect students were creating their own text book. Notebooks were permitted in all tests. The use of notebooks supported students to reflect on the mathematics, i.e., think meta-cognitively, and encouraged them to focus on the underlying principles.

The emphasis on group work has made significant contribution to the collaborative climate of the classes. The class was divided into groups of about 6 students, which were self selected for the second half of the year. The collaborative climate was further enhanced by the team teaching approach used in which such cooperative behaviour has been modelled by the teachers.

Communication of mathematics was also a focus. Given the investigative nature of the majority of the work, the presentation and sharing of findings was highly valued and two main activities formed the central part of my pedagogical approach. There were Unseen Orals and Public Presentation Pieces (PPP). Unseen orals involved students presenting a solution to a problem to their peers orally with the aid of a whiteboard. These oral presentations provided the means for students to demonstrate their understanding and communication skills. The PPPs followed on from those in MAT, but in this case it was a group solution that was displayed. This may have been written up and done by an individual, some members of the group or all members.

A typical week of Mathematical Studies in term 3 consisted of three sessions: one working on the Investigations, one on an Unseen Oral and the last a quiz.

A set of Investigative Activities was the core work. One set was generally the basis for a week’s work. Each student was expected to do all the investigations. Further, each investigation was assigned to one or more groups for the preparation and display of a solution.

Monday was a 50 minute session to work on the Investigations. Some students would be working on the Investigations from the previous week and their PPP. These were posted on a classroom noticeboard before the end of the session. This would give individual students the opportunity to check their

own work and to assist with their notebook entry. Other students would be starting work on the next set of Investigations, asking questions about the quiz or working on an extension activity.

Wednesday was a 100 minute session and began with an unseen oral. Typically examination style problems were compiled and given out to students. Each group was assigned a particular problem. Students then worked for 10 to 15 minutes individually. This was followed by group work. At this time the students developed their group solution (this part of the session often extended to 20 minutes or more and was accompanied a buzz of activity). To complete the activity, a member of each group then presented their solution orally using the whiteboard. Their audience consisted of their own group, a teacher and two other groups who had concentrated on different problems. A high level of engagement was demonstrated by the students in this activity. The last part of the session was used to continue work on the Investigations.

Friday, again a 50 minute session, was often a quiz. The quiz problems would usually involve similar problems to the unseen orals or to earlier investigations. The quizzes gave students the opportunity to check on their understanding of and ability to apply the concepts involved in the week's activities in a time restricted activity. At the end of each topic we had a test.

There was very little time spent by staff talking to the whole group. Rather our teaching was occurring in individual or small group situations where we were responsive to the students' concerns. That is, the role of the teacher was more a facilitator rather than a broadcaster of knowledge.

Most of the students were able to function effectively in this environment. Some were largely independent, using peer support to clarify and develop their understandings. Others relied more extensively on staff for direction and support. With so much opportunity and support for group work the classroom atmosphere was collaborative and informal.

Connection to ASMS T&L handbook

Students increasingly directing their own learning and learning independently

The delivery of the courses through sets of activities and problems provide opportunities for students to learn independently and in a variety of learning styles. Through the provision of core and extension investigations students direct their learning according to their interest, capability, goals and current workload. That is, they are supported to make informed decisions about their learning and to direct their own learning.

Learning which occurs as part of a process of constructing knowledge

This occurs by engaging in activities that present problem contexts where the mathematics is implicit. These contexts then provide reasons for the development of new mathematical tools and ideas to solve the problems. In this process students can develop their understanding of the mathematics. Upon

completion of the investigation, students are expected to make entries in their notebook. Such entries encapsulate the student's understandings of the key ideas which they have developed in undertaking the investigation.

Learning which involves learners communicating their questions, intuitions, conjectures, reasons, explanations and ideas

The extensive use of group work encourages collaboration and the communication of questions, intuitions, conjectures, reasons, explanations and ideas. Students are seen to be doing this consistently when working in their groups. Their learning is refined as they freely discuss their ideas and argue a case until they come to a shared understanding. It is a common occurrence to see groups of students discussing mathematics in the school. At times such discussions have become quite animated, the students clearly strongly engaged with the mathematics and wishing to communicate with their peers.

Learning which involves developing knowledge, skills and dispositions to think and act in ways which determine individual effort, the setting of personal goals and self awareness

The collaborative atmosphere and emphasis on students being responsible for their own learning supports students setting their own goals and being aware of their progress towards them. For me, as the teacher, I found it necessary to resist the temptation to take these decisions away from students. After all I know what they need to do to maximise their result. Such presumptuous attitudes reduce the opportunities for students to take responsibility and as such are likely to inhibit future learning, which is something that we are seeking to encourage. It is important to support, encourage and guide rather than to predigest the knowledge into small discrete chunks that are readily used but without fitting into a broader context.

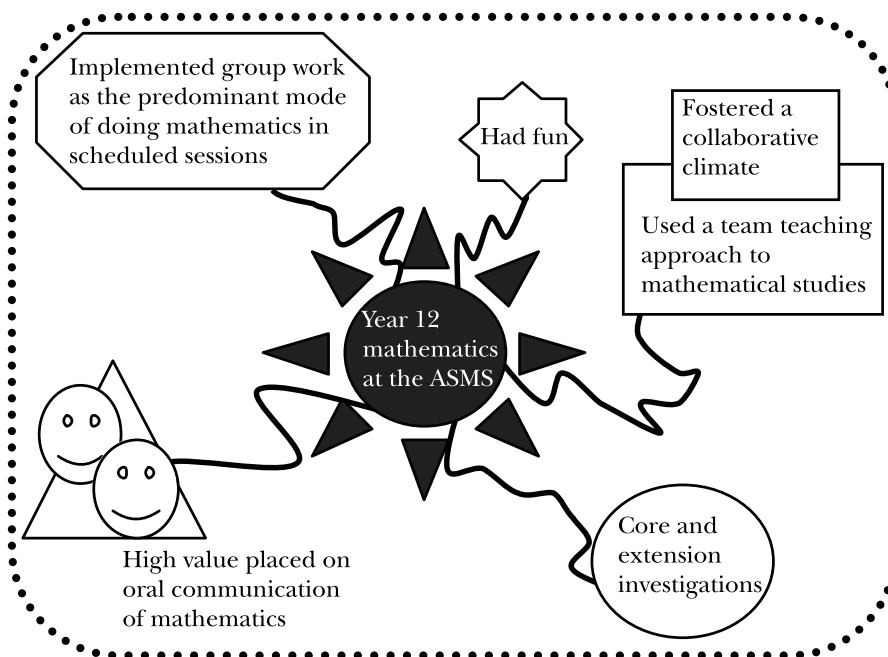


Figure 1. Concept map of the components of my approach to teaching Year 12 mathematics.

Table 1. Explicit and implicit beliefs with resultant pedagogical practices across the identified elements related to my teaching and student learning

	FORMER	CURRENT
SYLLABUS CONTENT		
<i>Explicit beliefs</i>	There is too much content in the Curriculum Statement. I feel rushed in getting through the syllabus.	There are relatively few key ideas and there is time to develop understandings of them. In the past I expected students to do busy work.
<i>Implicit beliefs</i>	The student is an empty vessel to be filled with objective knowledge.	The detail becomes easier/obvious when it fits within a framework.
<i>Pedagogical practices</i>	I do a lot of chalk and talk, I teach to the middle of the class. I push the students through the course.	I provide learning opportunities, I encourage peer mentoring and discussion. I encourage collaboration.
ACCOUNTABILITY		
<i>Explicit beliefs</i>	My students' results are a reflection of my professional abilities. That is if they do poorly, by school or my standards, then I feel I have failed.	Students construct their own understandings. When they engage with the course and me then they will do OK. My role is to provide opportunities for learning.
<i>Implicit beliefs</i>	The teacher has the primary responsibility for the learning of each student. Students cannot be trusted to make the right decisions, i.e., to do what teachers/parents/society want them to.	Students have the primary responsibility for their own learning. This will encourage the development of lifelong learners.
<i>Pedagogical practices</i>	Highly teacher directed. Bullying and cajoling, i.e., the use of sticks and carrots to motivate students, e.g., "You need to do this as it is for assessment."	Allow space for individual approaches to learning. Be flexible develop a cooperative classroom culture. Accept and allow students to take responsibility for their own learning.
PEDAGOGY		
<i>Explicit beliefs</i>	Students will only learn the stuff required to get good marks if I explicitly tell (teach) them.	Students construct their own understandings.
<i>Implicit beliefs</i>	Students are not naturally curious. Year 12 is hard work.	Understand key ideas and the rest follows so much more easily.
<i>Pedagogical practices</i>	I do a lot of chalk and talk, I question them to probe their level of understanding. I explicitly tell them what they need to know .	I provide learning opportunities, I encourage peer mentoring and social interaction. I encourage sharing of knowledge.
ENGAGEMENT		
<i>Explicit beliefs</i>	I know that many students will not get it the first time through.	I know that many students will not get it the first time.
<i>Implicit beliefs</i>	This is not a reflection of my teaching, rather it is a reflection of the students intellectual abilities or motivation.	It takes time to modify existing schema.
<i>Pedagogical practices</i>	I set lots of repetitive exercises. I use drill and practice approaches. I repeat the same stuff in chalk and talk sessions.	I allow multiple approaches and time to engage.
RESULTS		
<i>Explicit beliefs</i>	Students will be prepared for the exam if I drill them and give them lots of practice of working under exam conditions. Students who can do it in the exam are those that can really do mathematics.	Students will be better prepared for life and further study if I encourage them to be independent learners.
<i>Implicit beliefs</i>	Life success depends upon a good Year 12 mark.	Being competent learners will be of greater benefit than a higher TER.
<i>Pedagogical practices</i>	Teacher bullies students into working using extrinsic motivation. Lots of tests and marks.	Teacher acts as facilitator.
AUTHORITIES		
<i>Explicit beliefs</i>	The text book is the Bible and past exam papers are the New Testament.	Working on problems is an excellent and efficient way of developing mathematics problem solving skills.
<i>Implicit beliefs</i>	Text book writers understand the course better than I do.	I have more understanding of my students than the writers of the text. I also understand the intention of the course.
<i>Pedagogical practices</i>	I rely on the text book and past exam papers to determine the depth of treatment required.	Curriculum documents provide guidance for designing materials and learning opportunities.
RELATIONSHIPS		
<i>Explicit beliefs</i>	I have a good relationship with my students. My enthusiasm and pleasure in doing mathematics will rub off on my students.	I have a good relationship with my students. When I manipulate students using extrinsic motivators then I diminish the relationship.
<i>Implicit beliefs</i>	Students will do more of what I want them to do when they like and respect me. It is my classroom. I act as a benevolent dictator.	A good relationship will lead to the best possible long term outcomes. It is our classroom. I act as a facilitator.
<i>Pedagogical practices</i>	I expect students to do as I wish.	I work with people.

My former beliefs and pedagogical practices for teaching Year 12 mathematics

This paper follows my learning journey and I have contrasted my former and current teaching practices in Table 1. Naturally, these have been and are grounded in ways of seeing the world, that is, I have constructed my own understandings of what it means to be a teacher of Year 12 mathematics.

In developing an understanding of these practices and the changes that have evolved I have used a series of explicit beliefs statements about my role in the classroom. These statements are the kind of thing that teachers say and I would have applied to myself at some point in my teaching career. I assert that these have underlying implicit beliefs about teaching and learning and I seek to give examples of the associated practices. These are contrasted with my current beliefs and associated practices.

I have categorised these statements as follows: syllabus content, accountability, teaching practices, engagement, results, authority and relationships for the purposes of this paper (see Table 1). Through contrasting my former and current beliefs and pedagogical practices, I was able to articulate how my current beliefs connect with my current pedagogical practices, the ASMS teaching and learning principles and constructivist approaches to learning.

Conclusion

I believe that:

- students engaged with the mathematics;
- did some wonderful mathematics in their solutions to investigations;
- students demonstrated the ability to learn much that they have not been explicitly taught by teachers;
- a constructivist pedagogy can be developed and used successfully for Year 12 mathematics;
- my role as a teacher is increasingly to work with others rather than tell others what they are to do, i.e., I am a facilitator of learning rather than a transmitter of my understandings;
- students are much better prepared for further study, have more positive attitudes to themselves as learners and the ability to determine appropriate career paths and further study choices for themselves as a result of the approaches adopted;
- students TER results are at least equivalent and probably improved as a result of the pedagogical approach used. This is suggested by an initial analysis of TER scores.

These statements are grounds for further planned research.

It has been a wonderful, exciting journey this year, being a part of a learning community, learning and doing mathematics. Not only has an exciting innovation in mathematics been trialled but a focus on underlying principles or key ideas has been demonstrated to be an effective pedagogy with wider application.

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